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Stata tip 58: _nl_ is not just for nonlinear models

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1 Introduction

The _nl_ command makes performing nonlinear least-squares estimation almost as easy as performing linear regression. In this tip, three examples are given where _nl_ is preferable to _regress_, even when the model is linear in the parameters.

2 Transforming independent variables

Using the venerable _auto_ dataset, suppose we want to predict the weight of a car based on its fuel economy measured in miles per gallon. We first plot the data:

```
. sysuse auto
. scatter weight mpg
```

Clearly, there is a negative relationship between _weight_ and _mpg_, but is that relationship linear? The engineer in each of us believes that the amount of gasoline used to go one mile should be a better predictor of weight than the number of miles a car can go on one gallon of gas, so we should focus on the reciprocal of _mpg_. One way to proceed would be to create a new variable, _gpm_, measuring gallons of gasoline per mile and then to use _regress_ to fit a model of _weight_ on _gpm_. However, consider using _nl_ instead:

```
. nl (weight = {b0} + {b1}/mpg)
(obs = 74)
Iteration 0: residual SS = 1.19e+07
Iteration 1: residual SS = 1.19e+07
```

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>32190898.6</td>
<td>1</td>
<td>32190898.6</td>
</tr>
<tr>
<td>Residual</td>
<td>11903279.8</td>
<td>72</td>
<td>165323.33</td>
</tr>
<tr>
<td>Total</td>
<td>44094178.4</td>
<td>73</td>
<td>604029.841</td>
</tr>
</tbody>
</table>

| weight | Coef. | Std. Err. | t    | P>|t| | [95% Conf. Interval] |
|--------|-------|-----------|------|------|----------------------|
| /b0    | 415.1925 | 192.5243 | 2.16 | 0.034 | 31.40241 798.9826 |
| /b1    | 51885.27 | 3718.301 | 13.95 | 0.000 | 4472.97 59297.56 |

Parameter _b0_ taken as constant term in model & ANOVA table
You can verify that $R^2$ from this model is higher than that from a linear model of weight on mpg. You can also verify that our results match those from regressing weight on gpm.

Here a key advantage of `nl` is that we do not need to create a new variable containing the reciprocal of mpg. When doing exploratory data analysis, we might want to consider using the natural log or square root of a variable as a regressor, and using `nl` saves us some typing in these cases. In general, instead of typing

\[
\texttt{. generate sqrtx = sqrt(x)} \\
\texttt{. regress y sqrtx}
\]

we can type

\[
\texttt{. nl (y = \{b0\} + \{b1\}*sqrt(x))}
\]

### 3 Marginal effects and elasticities

Using `nl` has other advantages as well. In many applications, we include not just the variable $x$ in our model but also $x^2$. For example, most wage equations express log wages as a function of experience and experience squared. Say we want to fit the model

\[
y_i = \alpha + \beta_1 x_i + \beta_2 x_i^2 + \epsilon_i
\]

and then determine the elasticity of $y$ with respect to $x$; that is, we want to know the percent by which $y$ will change if $x$ changes by one percent.

Given the interest in an elasticity, the inclination might be to use the `mfx` command with the `eyex` option. We might type

\[
\texttt{. generate xsq = x^2} \\
\texttt{. regress y x xsq} \\
\texttt{. mfx compute, eyex}
\]

These commands will not give us the answer we expect because `regress` and `mfx` have no way of knowing that `xsq` is the square of `x`. Those commands just see two independent variables, and `mfx` will return two “elasticities”, one for $x$ and one for `xsq`. If $x$ changes by some amount, then clearly $x^2$ will change as well; however, `mfx`, when computing the derivative of the regression function with respect to $x$, holds `xsq` fixed!

The easiest way to proceed is to use `nl` instead of `regress`:

\[
\texttt{. nl (y = \{a\} + \{b1\}*x + \{b2\}*x^2), variables(x)} \\
\texttt{. mfx compute, eyex}
\]

Whenever you intend to use `mfx` after `nl`, you must use the `variables()` option. This option causes `nl` to save those variable names among its estimation results.
4 Constraints

\texttt{n1} makes imposing nonlinear constraints easy. Say you have the linear regression model

\[ y_i = \alpha + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + \epsilon_i \]

and for whatever reason you want to impose the constraint that \( \beta_2 \beta_3 = 5 \). We cannot use the \texttt{constraint} command in conjunction with \texttt{regress} because \texttt{constraint} only works with linear constraints. \texttt{n1}, however, provides an easy way out. Our constraint implies that \( \beta_3 = 5/\beta_2 \), so we can type

\[
\texttt{n1 (y = (a) + (b1)*x1 + (b2=1)*x2 + (5/(b2))*x3)}
\]

Here we initialized \( \beta_2 \) to be 1 because if the product of \( \beta_2 \) and \( \beta_3 \) is not 0, then neither of those parameters can be 0, which is the default initial value used by \texttt{n1}. 