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From the help desk: Seemingly unrelated regression with unbalanced equations

Allen McDowell
StataCorp

Abstract. This article demonstrates how to estimate the parameters of a system of seemingly unrelated regressions when the equations are unbalanced, i.e., when the equations have an unequal number of observations. With estimators that require the data to be in wide format, such as Stata’s `sureg`, the equations must be balanced. Any additional observations that are available for some equations, but not for all, are discarded, potentially resulting in a loss of efficiency. Reshaping and scaling the data allows us to use Stata’s `xtgee` command to fit the model and obtain estimates utilizing all the available data. The resulting estimator is potentially more efficient when the equations are unbalanced.

Keywords: st0079, SUR, seemingly unrelated regression, unbalanced equations, generalized estimating equations

1 Introduction

Fitting a system of equations via seemingly unrelated regression (SUR) with Stata’s `sureg` or `reg3` commands will result in a loss of information if the number of observations is not the same for all equations. The `xtgee` command provides an alternative estimator that can use all the available information, and for normally distributed data, `xtgee’s` iteratively reweighted least-squares estimator is equivalent to maximum likelihood.

2 Preparing the data

To help you visualize the steps needed to prepare your data, note that a system of three seemingly unrelated equations that can be written as

\[
\begin{align*}
y_1 &= X_1\beta_1 + \epsilon_1 \\
y_2 &= X_2\beta_2 + \epsilon_2 \\
y_3 &= X_3\beta_3 + \epsilon_3
\end{align*}
\]

can also be written as one superequation

\[
\begin{bmatrix}
y_1 \\
y_2 \\
y_3
\end{bmatrix} =
\begin{bmatrix}
X_1 & 0 & 0 \\
0 & X_2 & 0 \\
0 & 0 & X_3
\end{bmatrix}
\begin{bmatrix}
\beta_1 \\
\beta_2 \\
\beta_3
\end{bmatrix} +
\begin{bmatrix}
\epsilon_1 \\
\epsilon_2 \\
\epsilon_3
\end{bmatrix}
\]

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st0079
This rearrangement of multiple linear equations into a single superequation generalizes for any number of equations. The key point of interest here is the block diagonal arrangement of the \( X_i \) and the fact that the off-diagonal blocks of the data matrix are populated with zeros. Assuming that you have a dataset in wide form, which is the most natural form for multiple-equation data, you need to take three steps to prepare the data before estimating with \texttt{xtgee}:

1. scale the data to account for equation-specific variances
2. reshape the data into long form
3. \texttt{tsset} the data

Let’s begin by simulating some data. First, we generate a set of normally distributed, correlated error terms for the three equations:

\begin{verbatim}
. set seed 2004
. mat c1 = (1, .9, .6 \ .9, 1, .75 \ .6, .75, 1)
. drawnorm e1 e2 e3, n(100) mean(0 0 0) sds(10 15 20) corr(c1)
\end{verbatim}

Next we generate a set of covariates:

\begin{verbatim}
. mat c2 = (1, -.6, -.009, .49, -.38, .002 \ -.6, 1, -.59, -.08, -.338 \ -.009, -.59, 1, -.18, -.11, .144 \ .49, -.608, -.18, 1, .46, .18 \ -.38, -.08, -.11, .46, 1, .004 \ -.338, .144, .18, .004, 1)
. drawnorm x11 x12 x21 x22 x31 x32, mean(10 5 19 20 15 12) sds(13 20 27 2 8 22) corr(c2)
\end{verbatim}

We can now generate the dependent variables:

\begin{verbatim}
. generate y1 = 100 + 15*x11 + .7*x12 + e1
. generate y2 = 75 + 25*x21 + 20*x22 + e2
. generate y3 = 50 + 15*x31 + 19*x32 + e3
\end{verbatim}

Finally, we set a few observations to missing so that the equations are unbalanced and generate a time variable to index the observations. The time variable will be used as the panel identifier when we \texttt{tsset} the data since it indexes contemporaneous observations across equations.

\begin{verbatim}
. replace y1 = . in 1/3
(3 real changes made, 3 to missing)
. replace y2 = . in 97/100
(4 real changes made, 4 to missing)
. replace x31 = . in 25
(1 real change made, 1 to missing)
. gen time = _n
. save wide, replace
\end{verbatim}

file wide.dta saved
Fitting a SUR model with `sureg` we get

```
. sureg (y1 x11 x12) (y2 x21 x22) (y3 x31 x32)
```

**Seemingly unrelated regression**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Obs</th>
<th>Parms</th>
<th>RMSE</th>
<th>&quot;R-sq&quot;</th>
<th>chi2</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>y1</td>
<td>92</td>
<td>2</td>
<td>9.16867</td>
<td>0.9973</td>
<td>123197.83</td>
<td>0.0000</td>
</tr>
<tr>
<td>y2</td>
<td>92</td>
<td>2</td>
<td>14.00425</td>
<td>0.9996</td>
<td>829710.24</td>
<td>0.0000</td>
</tr>
<tr>
<td>y3</td>
<td>92</td>
<td>2</td>
<td>17.88595</td>
<td>0.9978</td>
<td>77775.54</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

|               | Coef. | Std. Err. | z    | P>|z| | [95% Conf. Interval] |
|---------------|-------|-----------|-----|------|----------------------|
| y1            |       |           |     |      |                      |
| x11           | 15.01983 | .0517867 | 290.03 | 0.000 | 14.91833 15.12133 |
| x12           | .7003301 | .0466798 | 15.00 | 0.000 | .6088394 .7918207 |
| _cons         | 98.10696 | 1.233664 | 79.52 | 0.000 | 95.68902 100.5249 |
| y2            |       |           |     |      |                      |
| x21           | 24.99282 | .0330073 | 757.19 | 0.000 | 24.92812 25.05751 |
| x22           | 19.7629 | .4412294 | 44.79 | 0.000 | 18.89811 20.62769 |
| _cons         | 77.81727 | 9.237519 | 8.42 | 0.000 | 59.71207 95.92248 |
| y3            |       |           |     |      |                      |
| x31           | 14.79411 | .1726112 | 85.71 | 0.000 | 14.4558 15.13242 |
| x32           | 19.00814 | .071608 | 265.45 | 0.000 | 18.8678 19.14849 |
| _cons         | 50.27515 | 3.244142 | 15.50 | 0.000 | 43.91675 56.63355 |

Notice that all three equations have only 92 observations in the estimation sample, so we lost a total of 24 observations. We lost three observations in each equation because \(y_1\) was missing in the first three observations; we lost four observations in each equation because \(y_2\) was missing in the last four observations, and we lost one observation in each equation because \(x_{31}\) was missing in the 25th observation. By rearranging our data into long form and fitting a single-equation panel-data model, we will be able to recover 16 of the missing observations, restricting the loss of information due to missing values to the specific equations from which the data are actually unobserved.

Before we can fit the same model using `xtgee`, we must first rescale the data. While `sureg` allows for unequal error variances across equations, `xtgee` assumes that errors are homoskedastic. To allow for unequal error variances, we can fit separate OLS regressions for each equation and rescale the variables for each equation using their respective regression root mean squared error.

```
. quietly regress y1 x11 x12
. foreach v of var y1 x11 x12 {
  2.       replace 'v' = 'v' / e(rmse)
  3. }
(97 real changes made)
(100 real changes made)
(100 real changes made)
. generate cons1 = 1/e(rmse)
. quietly regress y2 x21 x22
```
A. McDowell

foreach v of var y2 x21 x22 {
    replace 'v' = 'v' / e(rmse)
}
(96 real changes made)
(100 real changes made)
(100 real changes made)
generate cons2 = 1/e(rmse)
quietly regress y3 x31 x32
foreach v of var y3 x31 x32 {
    replace 'v' = 'v' / e(rmse)
}
(100 real changes made)
(99 real changes made)
(100 real changes made)
generate cons3 = 1/e(rmse)
save rescaled, replace
file rescaled.dta saved

Note that a scaled constant was generated for each equation. These scaled constants will be used instead of a single intercept in the `xtgee` version of the model.

We must now save the data for each equation in a separate dataset, perform some data manipulations, and append the three datasets together. However, before we proceed, a short digression on the mechanics of appending datasets is called for; the reason will become readily apparent as we progress.

3 A digression on appending datasets

Suppose that you have two datasets, A and B. In dataset A, you have two variables, var1 and var2; in dataset B, you have two variables, var1 and var3. For simplicity, suppose that there are just two observations in each dataset.

```
. use A
. list

<table>
<thead>
<tr>
<th>var1</th>
<th>var2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>.</td>
<td>2</td>
</tr>
</tbody>
</table>

. use B, clear
. list

<table>
<thead>
<tr>
<th>var1</th>
<th>var3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>
```

It is important that you know what to expect when we append these two datasets together.
From the help desk

Notice the missing values that are generated. In the case of var1, which was common to both datasets, the observations from B were stacked under the observations from A. The missing value that was present in A is still present in the appended dataset. Now look at var2 and var3 in the appended dataset. var2 was unique to A, and when the observations from B were appended, missing values were generated for var2 in the observations that originated in B. Similarly, var3 was unique to B, and when the observations from B were appended, missing values were generated for var3 in the observations from A. Before we can fit the model with xtgee, we must be able to distinguish between the missing values, such as those for var1 in the appended dataset, that are missing because the data were actually missing in one of the original datasets and those missing values that were generated for var2 and var3 because the respective variables were absent from one of the original datasets. We will want to recode the missing values of the latter type to zeros, leaving the missing values of the former type as missing in the appended dataset.

4 Preparing the data, continued

After saving the data from each equation in a separate dataset, we will rename the dependent variables so that they share a common variable name. We will then append all the datasets together. When we do so, blocks of missing values will be generated, just as demonstrated above. These missing values will then be recoded to zeros, thus forming the off-diagonal blocks of zeros that were described above when we converted the multiple equations into a single superequation. As discussed in A digression on appending datasets, when recoding the missing values that are generated by appending the datasets together, we must take care not to recode the missing values that were present in the original dataset. As we break up the original wide dataset into individual equation-specific datasets, we will generate variables that indicate if an observation has any missing values. These variables should share a common name in all the equation-specific datasets so that, when they are appended together, we have a single variable that marks the observations for which there were missing values to be retained in the appended dataset. We also need to generate a new variable for each dataset that identifies the equation from which the data originates; these variables should also share a common name so that, when the datasets are appended, we have a single variable that identifies the equation of origin for each observation. This new identification variable,
along with the time variable from the original dataset, will allow us to properly \texttt{tsset} the data for \texttt{xtgee}.

\begin{verbatim}
. preserve  
. keep y1 x11 x12 cons1 time  
. mark sample  
. markout sample y1 x11 x12 cons1  
. rename y1 y  
. gen id = 1  
. save data1, replace  
file data1.dta saved  
. restore  
. preserve  
. keep y2 x21 x22 cons2 time  
. mark sample  
. markout sample y2 x21 x22 cons2  
. rename y2 y  
. gen id = 2  
. save data2, replace  
file data2.dta saved  
. restore  
. preserve  
. keep y3 x31 x32 cons3 time  
. mark sample  
. markout sample y3 x31 x32 cons3  
. rename y3 y  
. gen id = 3  
. save data3, replace  
file data3.dta saved  
. restore  
. clear  
. use data1  
. append using data2  
. append using data3  
. mvencode x* cons* if sample, mv(0)  
  x11: 195 missing values recoded  
x12: 195 missing values recoded  
x21: 196 missing values recoded  
x22: 196 missing values recoded  
x31: 193 missing values recoded  
x32: 193 missing values recoded  
cons1: 195 missing values recoded  
cons2: 196 missing values recoded  
cons3: 193 missing values recoded  
. tsset time id  
  panel variable: time, 1 to 100  
time variable: id, 1 to 3
\end{verbatim}
Note that, when the data is `tsset`, the panel-identification variable and the time variable have reversed the roles that they usually perform. Since we are interested in modeling the contemporaneous correlation across equations, time is the relevant panel identifier, and the equation identifier indexes the repeated measures within panel.

## 5 Fitting the SUR model with `xtgee`

We are now ready to fit the SUR model using `xtgee`. The appropriate specification requires that we fit the model with a Gaussian family, an identity link, and since the SUR model imposes no structure on the correlation matrix, an unstructured within-group correlation structure. Since we also have generated rescaled constants for each equation, we must explicitly include them in our model and specify the `noconstant` option.

```
.xtgee y x* cons*, family(gaussian) link(identity) corr(unstructured) noconstant
```

Iteration 1: tolerance = .1167785
Iteration 2: tolerance = .00171051
Iteration 3: tolerance = 5.458e-06
Iteration 4: tolerance = 3.981e-07

GEE population-averaged model

| Coef. | Std. Err. | z    | P>|z| | [95% Conf. Interval] |
|-------|-----------|------|------|----------------------|
| x1    | 15.00825  | 0.057663 | 260.27 | 0.000 | 14.89523  | 15.12127 |
| x12   | 0.6771566 | 0.0474135 | 14.28  | 0.000 | 0.5842279 | 0.7700853 |
| x21   | 25.00623  | 0.032086  | 779.35 | 0.000 | 24.94355  | 25.06912 |
| x22   | 19.88076  | 0.4328133 | 45.93  | 0.000 | 19.03246  | 20.72906 |
| x31   | 14.75204  | 0.1648838 | 89.47  | 0.000 | 14.42887  | 15.07521 |
| x32   | 19.01571  | 0.066133  | 287.54 | 0.000 | 18.88609  | 19.14533 |
| cons1 | 98.25454  | 1.277547  | 76.91  | 0.000 | 95.75059  | 100.7585 |
| cons2 | 75.11264  | 0.91052   | 83.34  | 0.000 | 74.35235  | 75.87294 |
| cons3 | 50.68025  | 3.110645  | 16.29  | 0.000 | 44.58355  | 56.77701 |

Wald chi2(8) = 2021334
Prob > chi2 = 0.0000

Inspection of the output indicates that we have accomplished our goal. Rather than losing 24 observations due to missing values, the `xtgee` estimator was able to fit the model losing only the 8 observations for which the data were actually unobserved.

### About the Author

Allen McDowell is Director of Technical Services at StataCorp.