

THE STATA JOURNAL

Editor

H. Joseph Newton
Department of Statistics
Texas A&M University
College Station, Texas 77843
979-845-8817; fax 979-845-6077
jnewton@stata-journal.com

Editor

Nicholas J. Cox
Department of Geography
Durham University
South Road
Durham City DH1 3LE UK
n.j.cox@stata-journal.com

Associate Editors

Christopher F. Baum
Boston College

Nathaniel Beck
New York University

Rino Bellocco
Karolinska Institutet, Sweden, and
Univ. degli Studi di Milano-Bicocca, Italy

Maarten L. Buis
Vrije Universiteit, Amsterdam

A. Colin Cameron
University of California–Davis

Mario A. Cleves
Univ. of Arkansas for Medical Sciences

William D. Dupont
Vanderbilt University

David Epstein
Columbia University

Allan Gregory
Queen's University

James Hardin
University of South Carolina

Ben Jann
ETH Zürich, Switzerland

Stephen Jenkins
University of Essex

Ulrich Kohler
WZB, Berlin

Frauke Kreuter
University of Maryland–College Park

Stata Press Editorial Manager

Stata Press Copy Editors

Jens Lauritsen
Odense University Hospital

Stanley Lemeshow
Ohio State University

J. Scott Long
Indiana University

Thomas Lumley
University of Washington–Seattle

Roger Newson
Imperial College, London

Austin Nichols
Urban Institute, Washington DC

Marcello Pagano
Harvard School of Public Health

Sophia Rabe-Hesketh
University of California–Berkeley

J. Patrick Royston
MRC Clinical Trials Unit, London

Philip Ryan
University of Adelaide

Mark E. Schaffer
Heriot-Watt University, Edinburgh

Jeroen Weesie
Utrecht University

Nicholas J. G. Winter
University of Virginia

Jeffrey Wooldridge
Michigan State University

Lisa Gilmore

Jennifer Neve and Deirdre Patterson

The *Stata Journal* publishes reviewed papers together with shorter notes or comments, regular columns, book reviews, and other material of interest to Stata users. Examples of the types of papers include 1) expository papers that link the use of Stata commands or programs to associated principles, such as those that will serve as tutorials for users first encountering a new field of statistics or a major new technique; 2) papers that go “beyond the Stata manual” in explaining key features or uses of Stata that are of interest to intermediate or advanced users of Stata; 3) papers that discuss new commands or Stata programs of interest either to a wide spectrum of users (e.g., in data management or graphics) or to some large segment of Stata users (e.g., in survey statistics, survival analysis, panel analysis, or limited dependent variable modeling); 4) papers analyzing the statistical properties of new or existing estimators and tests in Stata; 5) papers that could be of interest or usefulness to researchers, especially in fields that are of practical importance but are not often included in texts or other journals, such as the use of Stata in managing datasets, especially large datasets, with advice from hard-won experience; and 6) papers of interest to those who teach, including Stata with topics such as extended examples of techniques and interpretation of results, simulations of statistical concepts, and overviews of subject areas.

For more information on the *Stata Journal*, including information for authors, see the web page

<http://www.stata-journal.com>

The *Stata Journal* is indexed and abstracted in the following:

- Science Citation Index Expanded (also known as SciSearch®)
- CompuMath Citation Index®

Copyright Statement: The *Stata Journal* and the contents of the supporting files (programs, datasets, and help files) are copyright © by StataCorp LP. The contents of the supporting files (programs, datasets, and help files) may be copied or reproduced by any means whatsoever, in whole or in part, as long as any copy or reproduction includes attribution to both (1) the author and (2) the *Stata Journal*.

The articles appearing in the *Stata Journal* may be copied or reproduced as printed copies, in whole or in part, as long as any copy or reproduction includes attribution to both (1) the author and (2) the *Stata Journal*.

Written permission must be obtained from StataCorp if you wish to make electronic copies of the insertions. This precludes placing electronic copies of the *Stata Journal*, in whole or in part, on publicly accessible web sites, file servers, or other locations where the copy may be accessed by anyone other than the subscriber.

Users of any of the software, ideas, data, or other materials published in the *Stata Journal* or the supporting files understand that such use is made without warranty of any kind, by either the *Stata Journal*, the author, or StataCorp. In particular, there is no warranty of fitness of purpose or merchantability, nor for special, incidental, or consequential damages such as loss of profits. The purpose of the *Stata Journal* is to promote free communication among Stata users.

The *Stata Journal*, electronic version (ISSN 1536-8734) is a publication of Stata Press. Stata and Mata are registered trademarks of StataCorp LP.

Stata tip 63: Modeling proportions

Christopher F. Baum
Department of Economics
Boston College
Chestnut Hill, MA
baum@bc.edu

You may often want to model a response variable that appears as a proportion or fraction: the share of consumers' spending on food, the fraction of the vote for a candidate, or the fraction of days when air pollution is above acceptable levels in a city. To handle these data properly, you must take account of the bounded nature of the response. Just as a linear probability model on unit record data can generate predictions outside the unit interval, using a proportion in a linear regression model will generally yield nonsensical predictions for extreme values of the regressors.

One way to handle this for response variables' values strictly within the unit interval is the logit transformation

$$y = \frac{1}{1 + \exp(-X\beta)}$$

which yields the transformed response variable y^*

$$y^* = \log\left(\frac{y}{1-y}\right) = X\beta + \epsilon$$

where we have added a stochastic error process ϵ to the model to be fitted. This transformation may be performed with Stata's `logit()` function. We can then use linear regression ([R] `regress`) to model y^* , the *logit transformation* of y , as a linear function of a set of regressors, X . If we then generate predictions for our model ([R] `predict`), we can apply Stata's `invlogit()` function to express the predictions in units of y . For instance,

```
. use http://www.stata-press.com/data/r10/census7  
(1980 Census data by state)  
. generate adu18p = pop18p/pop  
. quietly tabulate region, generate(R)  
. generate marrate = marriage/pop  
. generate divrate = divorce/pop  
. generate ladu18p = logit(adu18p)
```

```
. regress ladultpop marrate divrate R1-R3
```

Source	SS	df	MS			
Model	.164672377	5	.032934475	Number of obs =	49	
Residual	.327373732	43	.007613343	F(5, 43) =	4.33	
				Prob > F =	0.0028	
				R-squared =	0.3347	
				Adj R-squared =	0.2573	
Total	.492046109	48	.010250961	Root MSE =	.08725	

ladultpop	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
marrate	-18.26494	7.754941	-2.36	0.023	-33.90427	-2.625615
divrate	8.600844	12.09833	0.71	0.481	-15.79777	32.99946
R1	.1192464	.0482428	2.47	0.017	.0219555	.2165373
R2	.0498657	.042209	1.18	0.244	-.0352569	.1349883
R3	.0582061	.0357729	1.63	0.111	-.0139368	.130349
_cons	.999169	.093568	10.68	0.000	.8104712	1.187867

```
. predict double ladultpophat, xb
. generate adultpophat = invlogit(ladultpophat)
. summarize adultpop adultpophat
```

Variable	Obs	Mean	Std. Dev.	Min	Max
adultpop	49	.7113268	.0211434	.6303276	.7578948
adultpophat	49	.7116068	.0120068	.6855482	.7335103

Alternatively, we could use Stata's grouped logistic regression ([R] **glogit**) to fit the model. This command uses the same transformation on the response variable, which must be provided for the number of positive responses and the total number of responses (that is, the numerator and denominator of the proportion). For example,

```
. glogit pop18p pop marrate divrate R1-R3
Weighted LS logistic regression for grouped data
```

Source	SS	df	MS			
Model	.129077492	5	.025815498	Number of obs =	49	
Residual	.261736024	43	.006086884	F(5, 43) =	4.24	
				Prob > F =	0.0032	
				R-squared =	0.3303	
				Adj R-squared =	0.2524	
Total	.390813516	48	.008141948	Root MSE =	.07802	

pop18p	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
marrate	-22.82454	8.476256	-2.69	0.010	-39.91853	-5.730537
divrate	18.44877	12.66291	1.46	0.152	-7.088418	43.98596
R1	.0762246	.0458899	1.66	0.104	-.0163212	.1687704
R2	-.0207864	.0362001	-0.57	0.569	-.0937909	.0522181
R3	.0088961	.0354021	0.25	0.803	-.062499	.0802912
_cons	1.058316	.0893998	11.84	0.000	.8780241	1.238608

These results differ from those of standard regression because **glogit** uses weighted least-squares techniques. As explained in [R] **glogit**, the appropriate weights correct for the heteroskedastic nature of ϵ has zero mean but variance equal to

$$\sigma_j^2 = \frac{1}{n_j p_j (1 - p_j)}$$

By generating those weights, where n_j is the number of responses in the j th category and p_j is the predicted value we computed above, we can reproduce the `glogit` results with `regress` by using analytic weights, as verified with the commands:

```
. generate glswt = adu1tpophat * (1 - adu1tpophat) * pop
. quietly regress ladu1tpop marrate divrate R1-R3 [aw=glswt]
```

In the case of these state-level census data, values for the proportion y must lie within the unit interval. But we often consider data for which the limiting values of zero or one are possible. A city may spend 0% of its budget on preschool enrichment programs. A county might have zero miles of active railway within its boundaries. There might have been zero murders in a particular town in each of the last five years. A hospital may have performed zero heart transplants last year. In other cases, we may find values of one for particular proportions of interest. Neither zeros nor ones can be included in the strategy above, as the logit transformation is not defined for those values.

A strategy for handling proportions data in which zeros and ones may appear as well as intermediate values was proposed by [Papke and Wooldridge \(1996\)](#). At the time of their writing, Stata's generalized linear model (`[R] glm`) command could not handle this model, but it has been enhanced to do so. This approach makes use of the logit link function (that is, the logit transformation of the response variable) and the binomial distribution, which may be a good choice of family even if the response is continuous. The variance of the binomial distribution must go to zero as the mean goes to either 0 or 1, as in each case the variable is approaching a constant, and the variance will be maximized for a variable with mean of 0.5.

To illustrate, consider an alternative dataset that contains zeros and ones in its response variable, `meals`: the proportion of students receiving free or subsidized meals at school.

(Continued on next page)

```
. use http://www.ats.ucla.edu/stat/stata/faq/proportion, clear
. summarize meals
```

Variable	Obs	Mean	Std. Dev.	Min	Max
meals	4421	.5188102	.3107313	0	1

```
. glm meals yr_rnd parented api99, link(logit) family(binomial) vce(robust) nolog
note: meals has noninteger values
```

```
Generalized linear models                No. of obs    =    4257
Optimization      : ML                   Residual df   =    4253
                                                Scale parameter =    1
Deviance          = 395.8141242           (1/df) Deviance = .093067
Pearson           = 374.7025759           (1/df) Pearson = .0881031
Variance function: V(u) = u*(1-u/1)      [Binomial]
Link function     : g(u) = ln(u/(1-u))    [Logit]
                                                AIC           = .7220973
                                                BIC           = -35143.61
```

	Coef.	Robust Std. Err.	z	P> z	[95% Conf. Interval]	
meals						
yr_rnd	.0482527	.0321714	1.50	0.134	-.0148021	.1113074
parented	-.7662598	.0390715	-19.61	0.000	-.8428386	-.6896811
api99	-.0073046	.0002156	-33.89	0.000	-.0077271	-.0068821
_cons	6.75343	.0896767	75.31	0.000	6.577667	6.929193

The techniques used above can be used to generate predictions from the model and transform them back into the units of the response variable. This approach is preferred to that of dropping the observations with zero or unit values, which would create a truncation problem, or coding them with some arbitrary value (“winsorizing”) such as 0.0001 or 0.9999.

Some researchers have considered using censored normal regression techniques such as `tobit` ([R] `tobit`) on proportions data that contain zeros or ones. However, this is not an appropriate strategy, as the observed data in this case are not censored: values outside the $[0, 1]$ interval are not feasible for proportions data.

One concern was voiced about proportions data containing zeros or ones.¹ In the context of the generalized tobit or “heckit” model ([R] `heckman`), we allow for limit observations (for instance, zero values) being generated by a different process than non-censored observations. The same argument may apply here, in the case of proportions data: the managers of a city that spends none of its resources on preschool enrichment programs have made a discrete choice. A hospital with zero heart transplants may be a facility whose managers have chosen not to offer certain advanced services.

In this context, the `glm` approach, while properly handling both zeros and ones, does not allow for an alternative model of behavior generating the limit values. If different factors generate the observations at the limit points, a sample selection issue arises. [Li and Nagpuranand \(2007\)](#) argue that selection issues arise in numerous variables of interest in corporate finance research. In a forthcoming article, Cook, Kieschnick, and

1. See, for instance, [McDowell and Cox \(2001\)](#).

McCullough (2008) address this issue for proportions of financial variables by developing what they term the “zero-inflated beta” model, which allows for zero values (but not unit values) in the proportion and for separate variables influencing the zero and nonzero values.²

References

- Cook, D. O., R. Kieschnick, and B. D. McCullough. 2008. Regression analysis of proportions in finance with self selection. *Journal of Empirical Finance*. In press.
- Li, K., and R. Nagpurnanand. 2007. Self-selection models in corporate finance. In *Handbook of Corporate Finance: Empirical Corporate Finance*, ed. B. E. Eckbo, chap. 2. Amsterdam: Elsevier.
- McDowell, A., and N. J. Cox. 2001. FAQ: How do you fit a model when the dependent variable is a proportion? <http://www.stata.com/support/faqs/stat/logit.html>.
- Papke, L. E., and J. M. Wooldridge. 1996. Econometric methods for fractional response variables with an application to 401(K) plan participation rates. *Journal of Applied Econometrics* 11: 619–632.

2. Their approach generalizes models fit with the beta distribution; user-written programs for that purpose may be located by typing `findit beta distribution`.